Loan Guarantees
Part IV - The Revised BSOPM - Problem Solution

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May 2017

In Part I of this series we developed a two state gurantee model and in Part II we developed a continuous time guarantee model. In Part III we calibrated the continuous time model to the market. In Part IV we will value an uncapped and capped guarantee using the continuous time model developed in Part II and calibrated in Part III.

Our Hypothetical Problem

We will continue to use the hypothetical problem from Part I...

Table 1: Model Parameters

Symbol	Description	Value	Reference
A_0	Enterprise value at time zero	1,366,700	Part I
C_0	Annualized cash flow at time zero	100,000	Hypothetical problem
D_T	Debt payoff amount at time T	500,000	Hypothetical problem
T	Guarantee term in years	3.0000	Hypothetical problem
Γ	Liquidation value adjustment	0.5308	Part III
α	Risk-free rate (continuous time)	0.0392	Part I
κ	Cost of capital (continuous time)	0.0979	Part I
μ	Cash flow growth rate (continuous time)	0.0247	Part I
p	Cumulative probability of default	0.1000	Hypothetical problem
ϕ	Dividend yield (continuous time)	0.0732	Part I
π	Recovery rate given default	0.4000	Hypothetical problem
σ	Return volatility	0.3858	Part III

We will use our model to answer the following question:

Question 1: What is the value of the guarantee without a cap?

Question 2: What is the value of the guarantee with a cap = \$250,000?

Generalized Valuation Equations

In Part II of this series we calculated the value of a guarantee at time zero. We want to alter the equations from Part II so that we can calculate the value of a guarantee at any time t. To do this we will redefine the variable t used in the Part II equations as follows...

$$T = \text{Term of the guarantee in years} \ \text{...and...} \ t = \text{Current time in years} \ \text{...where...} \ 0 \le t \le T \tag{1}$$

Using the definitions in Equation (1) above we will redefine the Part II equations for d_1, d_2, d_3 and d_4 as follows...

$$\begin{aligned}
d_1 &= \left[\ln\left(\frac{D_T}{A_t}\right) - \left(\alpha - \phi - \frac{1}{2}\,\sigma^2\right)(T-t)\right] \Big/ \sigma\sqrt{T-t} \\
d_2 &= d_1 - \sigma\sqrt{T-t} \\
d_3 &= \left[\ln\left(\frac{D_T - CAP}{\Gamma A_t}\right) - \left(\alpha - \phi - \frac{1}{2}\,\sigma^2\right)(T-t)\right] \Big/ \sigma\sqrt{T-t} \\
d_4 &= d_3 - \sigma\sqrt{T-t}
\end{aligned} \tag{2}$$

Using the definitions in Equation (1) above we will define the functions $f(\alpha,t)$ and $g(\phi,t)$ as follows...

$$f(\alpha,t)=\mathrm{Exp}\left\{-\alpha\,(T-t)\right\}\ \text{...and...}\ g(\phi,t)=\mathrm{Exp}\left\{-\phi\,(T-t)\right\} \tag{3}$$

Using Equations (1), (3) and (2) above we can rewrite the equation for the value of an uncapped guarantee from Part II as...

$$G_t=D_T\,f(\alpha,t)\,CND\left[d_1\right]-\Gamma A_t\,g(\phi,t)\,CND\left[d_2\right] \tag{4}$$

Using Equations (1), (3) and (2) above we can rewrite the equation for the value of a capped guarantee from Part II as...

$$G_t=D_T\,f(\alpha,t)\Big(CND\left[d_1\right]-CND\left[d_3\right]\Big)-\Gamma A_t\,g(\phi,t)\Big(CND\left[d_2\right]-CND\left[d_4\right]\Big)+CAP\,f(\alpha,t)\,CND\left[d_3\right] \tag{5}$$

Guarantee Model Parameter Values

Using Equation (3) above and the parameters from Table 1 above the value of the model parameter $f(\alpha,t)$ at time zero is...

$$f(\alpha,t)=\mathrm{Exp}\left\{-0.0392\times 3\right\}=0.8890 \tag{6}$$

Using Equation (3) above and the parameters from Table 1 above the value of the model parameter $g(\phi,t)$ at time zero is...

$$g(\phi,t)=\mathrm{Exp}\left\{-0.0732\times 3\right\}=0.8029 \tag{7}$$

Using Equation (2) above and the parameters from Table 1 above the value of the model parameter d_1 at time zero is...

$$d_1=\left[\ln\left(\frac{500000}{1366700}\right)-\left(0.0392-0.0732-\frac{1}{2}\times 0.3858^2\right)\times 3\right]\Big/(0.3858\times\sqrt{3})=-1.0183 \tag{8}$$

Using Equations (2) and (8) above and the parameters from Table 1 above the value of the model parameter d_2 at time zero is...

$$d_2=-1.0183-0.3858\times\sqrt{3}=-1.6865 \tag{9}$$

Using Equation (2) above and the parameters from Table 1 above the value of the model parameter d_3 at time zero is...

$$d_3=\left[\ln\left(\frac{500,000-250,000}{0.5308\times 1,366,700}\right)-\left(0.0392-0.0732-\frac{1}{2}\times 0.3858^2\right)\times 3\right]\Big/0.3858\times\sqrt{3}=-1.1077 \tag{10}$$

Using Equations (2) and (10) above and the parameters from Table 1 above the value of the model parameter d_4 at time zero is...

$$d_4=-1.1077-0.3858\times\sqrt{3}=-1.7759 \tag{11}$$

Note that the Excel equivalent to the cumulative normal distribution function is as follows...

$$CND(Z)=NORMSDIST(Z) \tag{12}$$

Answers To Our Hypothetical Problem

Question 1: What is the value of the guarantee without a cap?

Using Equation (4) above, the model parameter values in Equations (7) through (12) above, and the model parameters in Table 1 above, the value of the uncapped guarantee is...

$$\begin{aligned}G_0&=500,000\times 0.8890\times CND[-1.0183]-0.5308\times 1,366,700\times 0.8029\times CND[-1.6865]\\&=500,000\times 0.8890\times 0.1543-0.5308\times 1,366,700\times 0.8029\times 0.0459\\&=41,869\end{aligned} \tag{13}$$

Question 2: What is the value of the guarantee with a cap = \$250,000?

Using Equation (5) above, the model parameter values in Equations (7) through (12) above, and the model parameters in Table 1 above, the value of the capped guarantee is...

$$
\begin{aligned}
G_0 = &\ 500,000 \times 0.8890 \times (CND[-1.0183] - CND[-1.1077]) - 0.5308 \times 1,366,700 \times 0.8029 \times \\
&(CND[-1.6865] - CND[-1.7759]) + 250,000 \times 0.8890 \times CND[-1.1077] \\
&= 500,000 \times 0.8890 \times (0.1543 - 0.1340) - 0.5308 \times 1,366,700 \times 0.8029 \times \\
&(0.0459 - 0.0379) + 250,000 \times 0.8890 \times 0.1340 \\
&= 34,148
\end{aligned} \tag{14}
$$

References

[1] Gary Schurman, *Loan Guarantees: The Two State Default Model,* April, 2017.

[2] Gary Schurman, *Loan Guarantees: The Revised BSOPM - Model Basics,* May, 2017.

[3] Gary Schurman, *Loan Guarantees: The Revised BSOPM - Model Calibration,* May, 2017.